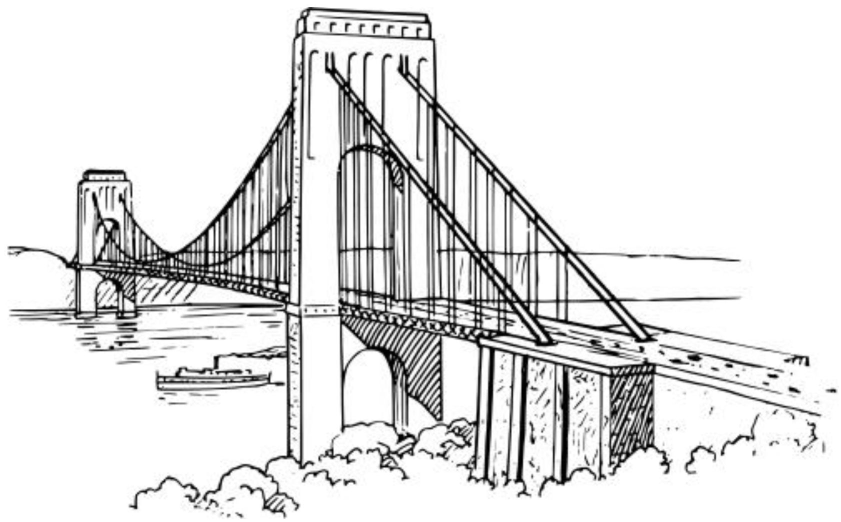
Mechanical properties of solids



Note : This PPT will NOT help you learn physics concepts. It is intended only as a quick revision of formulas, definitions, theorems and concepts before examinations. No physics can be learnt just by watching a few videos or going through a few slides of PPT.

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- **Deforming force**
  A force that tends to cause a change the dimensions (shape/size) of a body is called deforming force.

- **Restoring force**
  Restoring force causes the body to restore its initial size and shape after the deforming force is removed.
  It is the force that is developed in the body when it is subjected to a strain.

- **Elasticity**
  Property of a material by which it regains its initial size/shape when deforming force is removed.
  Eg: Quartz

- **Plasticity (or inelasticity)**
  Property of a body by which it does not regain its initial size/shape when the deforming force is removed.
  Eg: clay

- **Strain:**

  It is defined as change in dimension per unit dimension of the body.
  It is a dimensionless quantity.
  It doesn't have any units

- **Stress :**

  It is defined as the restoring force per unit area.

  Its dimensional formula is [$M^1L^{-1}T^{-2}$]

  Its SI unit is Nm^{-2}

- **Hooke's law**:

  Within elastic limit, ratio of stress to strain is constant.

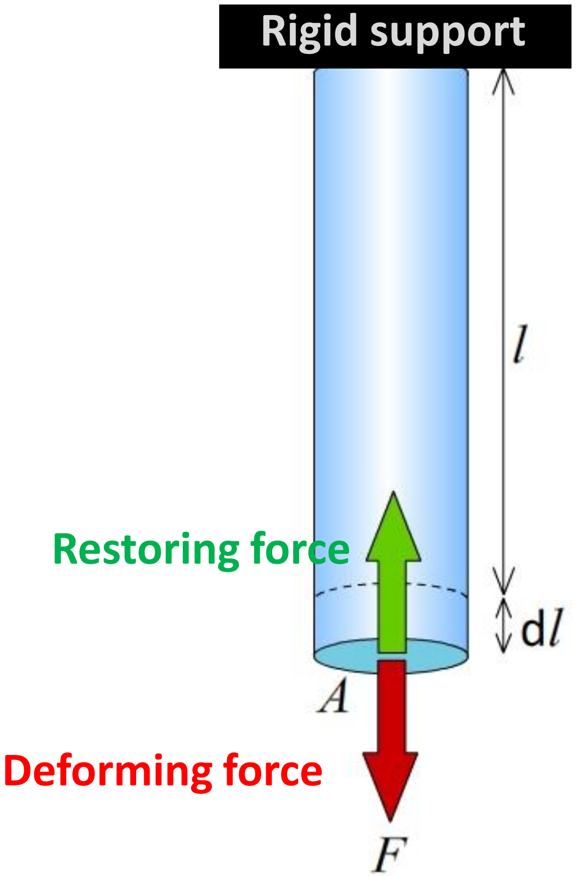
  The constant is, in general, called modulus of elasticity.

  (SI unit of modulus of elasticity is same as that of stress i.e. Nm^{-2})

General outline of study : Deforming force applied on a body causes a strain in it. This strain results in a stress in the body. Depending on nature of strain and the resulting stress, within elastic limit, there are three different kinds of moduli of elasticity.

Longitudinal strain, stress & Young's modulus

Consider a wire of length l suspended from a rigid support and subjected to a deforming force



Longitudinal strain : Change in length per unit length

$$\boxed{\text{strain} = \frac{\mathrm{d}l}{l}}$$

Longitudinal stress : Restoring force per unit area

$$\boxed{\text{strain} = \frac{F}{A}}$$

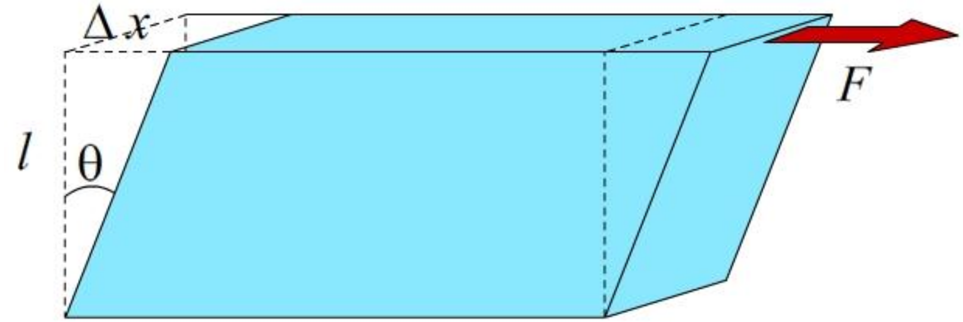
Young's modulus (Y) :
Ratio of longitudinal stress to longitudinal strain.

$$\boxed{Y = \frac{F}{A} \times \frac{l}{\mathrm{d}l}}$$

Tensile stress : when body is elongated
Compressive stress : when body is compressed

Shearing strain, stress & rigidity (or shear) modulus

Consider a cuboidal body placed on a horizontal surface with its lower end fixed to the ground. Let the deforming force be applied parallel to the top surface.



Shearing strain :

Ratio of relative displacement of opposite faces of an object per unit distance between the faces.

$$\text{shearing strain} = \frac{\Delta x}{l} = \tan(\theta)$$

Shearing (or tangential) stress :

Restoring force per unit area developed tangential to the surface.

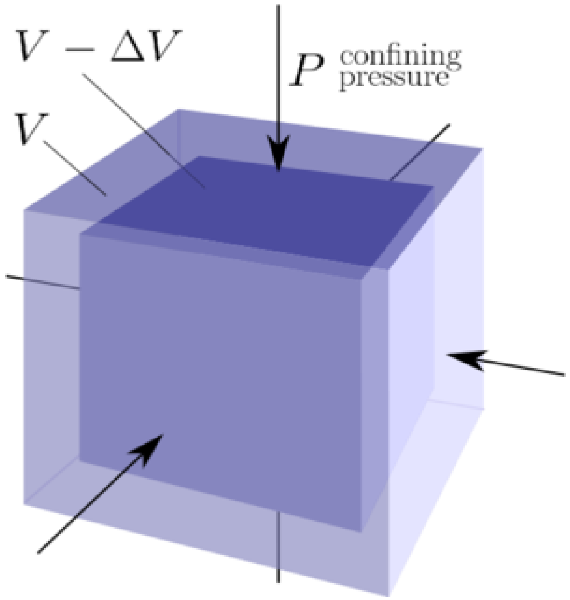
$$\text{stress} = \frac{F}{A}$$

Rigidity (or shearing) modulus : Ratio of shearing stress to shearing strain.

$$\eta = \frac{F}{A} \times \frac{l}{\Delta x}$$

Volume or bulk strain, bulk stress & bulk modulus

Consider a body subjected to normal force all over its surface. This deforming force results in compression of the body resulting in change in its volume.



Volume strain :

Ratio of change in volume to original volume.

$$\text{volume strain} = \frac{\Delta V}{V}$$

Bulk stress :

Restoring force developed per unit area .

$$\text{stress} = \frac{F}{A}$$

Bulk modulus :

Ratio of bulk stress to bulk strain.

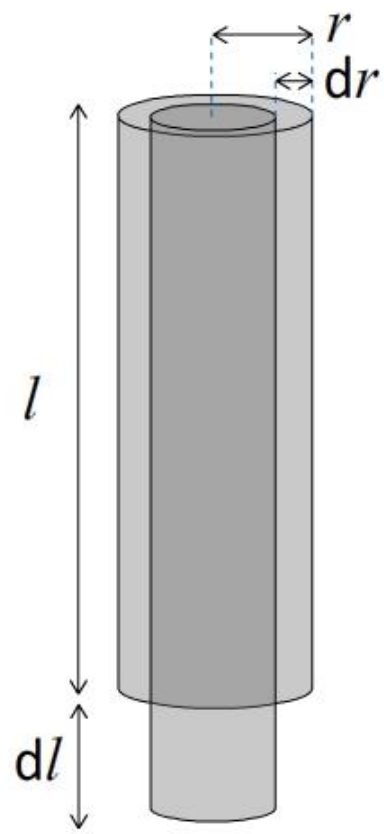
$$K = -P \times \frac{V}{\mathrm{d}V}$$

Compressibility : Reciprocal of bulk modulus.

Poison's ratio (σ)

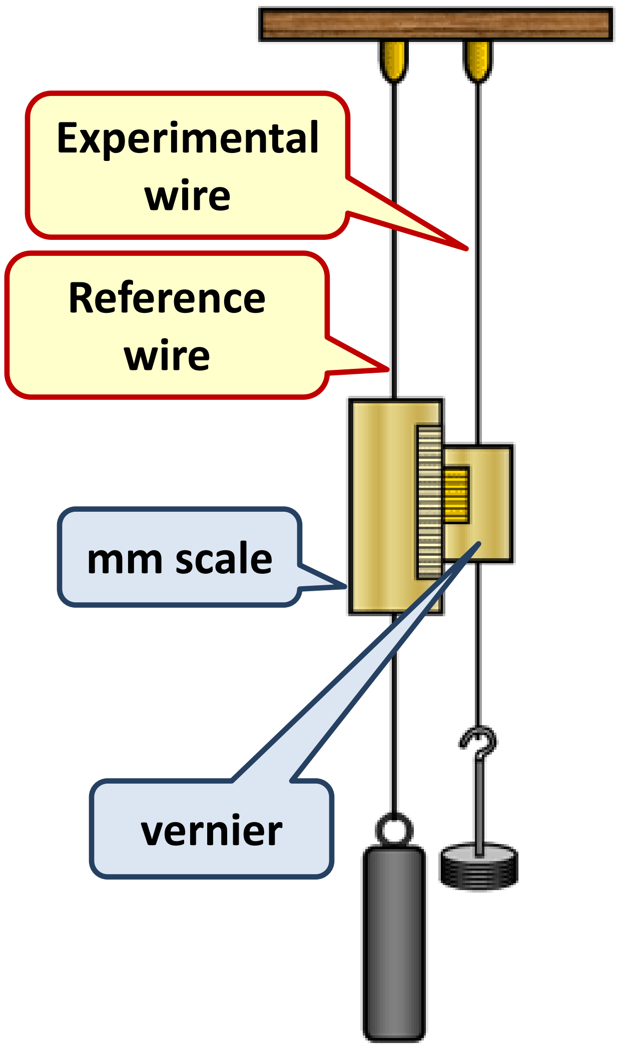
- It is defined as the ratio of lateral strain to longitudinal strain.
- It is a dimensionless quantity

$$\sigma = \frac{\mathrm{d}r}{r} \times \frac{l}{\mathrm{d}l}$$



Measurement of Young's modulus

- Both the wires are of identical dimensions.
- Both the wires are given an initial load that helps to keep the wires straight.
- The reference wire is connected to mm scale.
- The experimental wire is connected to vernier
- Weights are gradually added to the experimental wire and its elongation is measured using the vernier.
- Let L be the initial length of the wire and r be its radius
- Let m be the mass that results in an elongation of ΔL

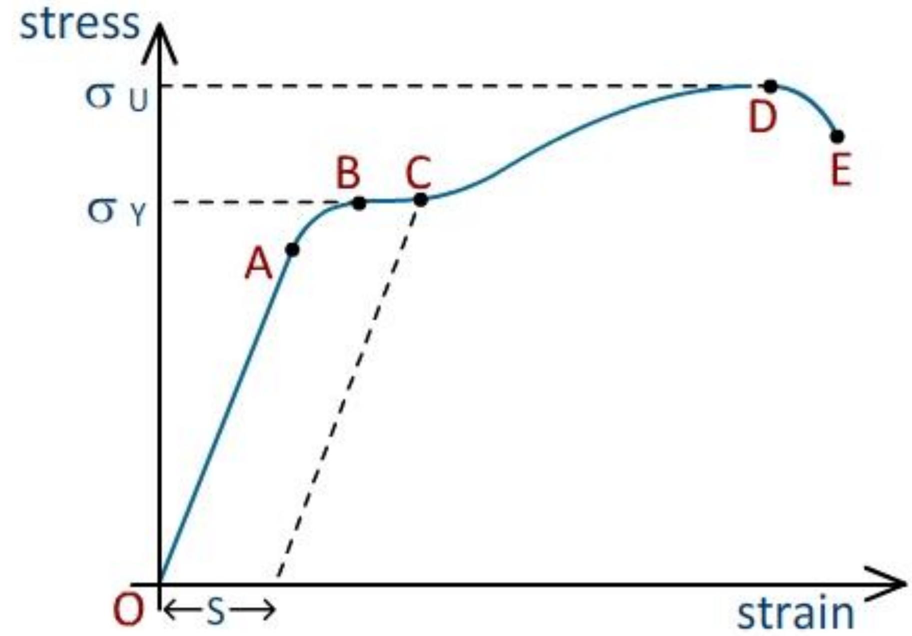


Young's modulus is given by the relation

$$\boxed{Y = \frac{mg}{\pi r^2} \times \frac{L}{\Delta L}}$$

Behaviour of a wire under increasing load

A wire is suspended from a rigid support. Free end of the wire is attached to a weight hanger. Load on wire is increased/decreased by changing the weights. A graph is plotted for stress as a function of strain.



Stress - strain curve for a metal

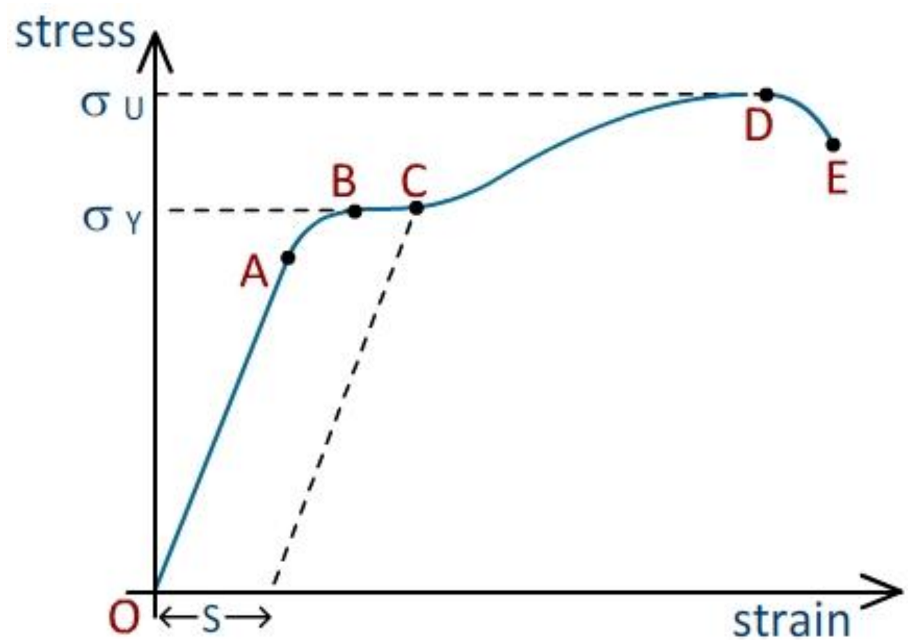
- Proportional limit (A): In this region stress is proportional to strain and Hooke's law is obeyed.
- Yield point or elastic limit (B) : The limit up to which the body retains elasticity. The corresponding stress is called yield strength.

Behaviour of a wire under increasing load

- Permanent set: Beyond yield point, certain amount of deformation is present in the wire even after deforming force is removed. This is called permanent set.

- Ultimate tensile strength (D) : Beyond this point, additional strain is produced even by a reduced load.

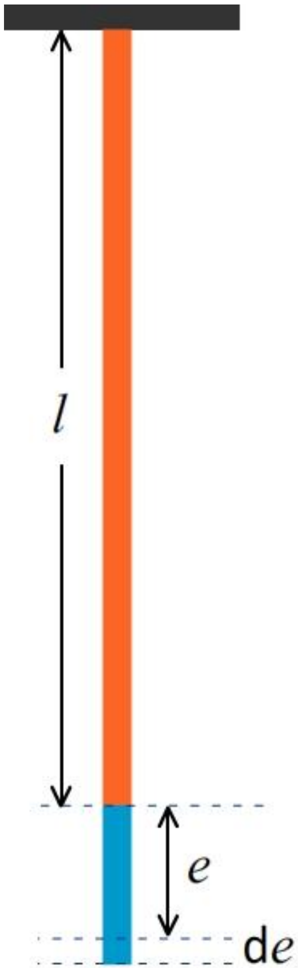
- If the ultimate strength and fracture points D and E are close, the material is said to be brittle. If they are far apart, the material is said to be ductile.



Stress - strain curve for a metal

Strain energy per unit volume

Consider a wire of initial length l, subjected to a deforming force resulting in an e in it.



Work done (dW) in causing a further small extension (de) of the wire is

$$dW = F\,de \quad \text{— i}$$

From Hooke's law we get

$$Y = \frac{F}{A}\frac{l}{e}$$

$$F = \frac{YAe}{l} \quad \text{— ii}$$

Substituting (ii) in equation (i)

$$dW = \frac{YAe}{l}de$$

Integrating the above expression

$$W = \frac{YA}{l}\int_0^e e^1\,de$$

$$W = \frac{YA}{l}\left[\frac{e^2}{2}\right]$$

This work is stored as elastic potential energy in the wire. Energy per unit volume is given by

$$\frac{U}{V} = \frac{YA}{l}\left[\frac{e^2}{2}\right]\times\frac{1}{Al}$$

$$\frac{U}{V} = \frac{1}{2}Y\frac{e^2}{l^2}$$

$$\boxed{\frac{U}{V} = \frac{1}{2}\text{stress} \times \text{strain}}$$